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How the West will be won: using Monte Carlo simulations to estimate the effects of NHL realignment

Abstract: The NHL has realigned its conferences and divisions, and starting with the 2013–2014 season the Eastern Conference features 16 teams and the Western Conference features 14. Yet because there are eight playoff spots available in both conferences, teams in the West have a 57% probability of making the playoffs, compared to just 50% for teams in the East. As a result we should expect that, on average, the last team to make the playoffs in the West will have a worse record than the last playoff team in the East. We call the difference in points earned by the 8th seed in each conference the “conference gap.” The purpose of this paper is to estimate the expected size of the conference gap under the new alignment. Using tens of thousands of simulated seasons, we demonstrate that the conference gap will be, on average, 2.74 points, meaning that Eastern Conference teams hoping to make the playoffs will have to win 1–2 games more than Western Conference playoff-hopefuls. We also show the 9th place team in the Eastern Conference has a better record than the 8th place Western team twice as often as the 9th best Western team has a better record than the East’s 8th best. Our findings inform questions about competitive balance and equity in the NHL.

Keywords: competitive balance; conference gap; conference realignment; Monte Carlo simulation; NHL.

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1 Introduction

When the Atlanta Thrashers relocated to Winnipeg following the 2010–2011 season, a realignment of the division structure in the NHL became a foregone conclusion. For reasons of competitive balance, Winnipeg could not remain in the Southeast Division. Thus, on March 14, 2013 the NHL

Board of Governors approved a plan for restructuring the league’s conferences and divisions. Beginning with the 2013–2014 season, the NHL was reorganized into an Eastern Conference featuring 16 teams and a Western Conference featuring just 14 (Hiebert 2013). Despite the imbalance in the size of the conferences, eight teams from the East and eight teams from the West now qualify for the playoffs.

Journalists have noted that such a system makes it more difficult for a team to make the playoffs in the Eastern Conference than in the Western Conference, since 50% of Eastern teams qualify each year while 57% of Western teams qualify (McCurdy 2013). This imbalance raises the question of how much more difficult will it be to make the playoffs in the East than in the West. Specifically, how many more points¹, on average, will the East’s 8th seed earn than the West’s 8th seed? If this difference, which we refer to as the “conference gap,” is zero then we can conclude that no team is receiving an unfair advantage when it comes to getting into the playoffs simply because of that team’s geographic location. If, however, the conference gap is not zero, we might question whether or not the system is fair.

This paper provides an answer to this question, even before we have data from enough seasons to make an empirical assessment. We begin by detailing the structural changes that have accompanied the realignment, specifically changes to scheduling and qualifying for the postseason. Next, we apply the new conference alignment to recent seasons and investigate how this would have altered the set of teams which qualified for the playoffs.

We then estimate the impact of realignment on future seasons using a Monte Carlo simulation which accounts for the new scheduling matrix and alignment (Pinsky and Karlin 2011). Monte Carlo methods are a common way for researchers to simulate games and seasons in hockey and

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¹ The NHL determines regular season standings based on “points.” A team receives two points for a win, one point for an overtime or shootout loss, and zero points for a loss in regulation. A team with 40 wins and 0 OT/SO losses will finish in the standings behind a team with 39 wins and 3 OT/SO losses, since the former team has 80 points and the latter has 81.

other sports. Rump (2006) simulates the Stanley Cup playoffs using a Markov chain. Rump (2008), Rudelius (2012), and Beaudoin (2013) each simulate sets of individual baseball games. Newton and Aslam (2009) simulate professional tennis tournaments. Pasteur and Janning (2011) use Monte Carlo methods to predict high school football seedings by simulating the regular season. Although these papers simulated game outcomes, to our knowledge this paper is the first time Monte Carlo methods have been used to assess the effect of a structural change in a professional sports league.

The results from our 10,000 simulated NHL seasons indicate that, when team talent is roughly evenly distributed between the two conferences, it will require 2.74 (SE: 0.06) more points on average to make the playoffs in the East than in the West. In other words, on average an Eastern Conference playoff-hopeful team will need to win about one or two more games than a Western Conference playoff-hopeful team. This finding has important implications for competitive balance in the NHL.

Previous scholarship has explored the effect of rule changes in professional sports on competitive balance. Quinn and Bursik (2007) show that MLB expansion and team relocation affects competitive equity in the short run, but that any imbalances resolve themselves quickly. Horowitz (1997) and Lee and Fort (2005) find similar short-term effects, but both show that the growth of TV revenue in baseball has had different long-term effects on teams depending on media market size.

Our research is one of the only studies to find long-term competitive balance effects of a structural change imposed by a professional sports league. Prior to 2013 MLB had a similar imbalance in the division makeup of the AL and NL. Its main motivation for restructuring the league in that year was to eliminate this imbalance (Rosenthal 2010). Blatt (2010) quantifies the effect of this division imbalance in terms of the probability that teams make the playoffs, but does not estimate the disparity in wins required to make the playoffs in a small division versus a large division. Our paper is the first attempt to quantitatively estimate the size of such a disparity in terms of wins and losses.

2 Realignment and structural changes

On May 31, 2011 it was announced that the Atlanta Thrashers had been sold to a group of Canadian investors who intended to move the team to Winnipeg, Manitoba. During

the following season, the team (now called the Winnipeg Jets) remained in the Southeast Division, with divisional opponents located in Washington, Raleigh, Tampa Bay, and Miami. As a result, the Jets traveled an average of 1500 miles to each of their twelve divisional road games. This geographic imbalance prompted realignment to become a key topic of discussion in NHL circles.

On March 14, 2013 the NHL Board of Governors gave the final approval to a realignment plan intended to alleviate geographic imbalances. The Jets switched to the Western Conference, and the Detroit Red Wings and Columbus Blue Jackets switched to the Eastern Conference. As a result, the two Western Conference divisions now feature seven franchises each, while the two Eastern Conference divisions feature eight.² Additionally, the scheduling matrix, which dictates the number of times each team plays the 29 others, has been altered in several ways. The bottom half of Table 1 shows these changes. Most notably, teams in the East and West will have about the same number of games against divisional opponents, but Eastern teams will have slightly fewer games against any particular divisional opponent (4.2) than Western teams (4.8).³

The other major change is the way in which teams qualify for the playoffs. In each of the four divisions, the three teams with the most points at the end of the season make the playoffs. The remaining two playoff teams in

Table 1 Changes resulting from realignment.

	Eastern conference	Western conference
New teams	Columbus Blue Jackets Detroit Red Wings	Winnipeg Jets
Number of divisions	2	2
Teams per division	8	7
Playoff teams	8	8
Division schedule	5 games×2 teams 4 games×5 teams 30 games	5 games×5 teams 4 games×1 team 29 games
Conference schedule	3 games×8 teams 24 games	3 games×7 games 21 games
Non-conference schedule	2 games×14 teams 28 games	2 games×16 teams 32 games

² Under the old alignment, each conference had three divisions of five teams each.

³ Note that in the Western Conference, one team from each division will play an extra game against each other and one fewer game against one divisional opponent.

Table 2 Changes to playoff picture if new conference structure were in place.

Season	Removed from playoffs				Added to playoffs			Diff. In:	
	Team (new conf)	Points	Wins	Playoff wins	Team (new conf)	Points	Wins	Points	Wins
'12-'13 ^a	Islanders (E)	55	24	2	Jets (W)	51	24	-4	0
'11-'12	Senators (E)	92	41	3	Flames (W)	90	37	-2	-4
'10-'11	Rangers (E)	93	44	1	Stars (W)	95	42	+2	-2
'09-'10	Canadiens (E)	88	39	9	Blues (W)	90	40	+2	+1
'08-'09	Rangers (E)	95	43	3	Predators (W)	88	40	-7	-3
'08-'09	Blue Jackets (E)	92	41	0	Wild (W)	89	40	-3	-1
'07-'08	Bruins (E)	94	41	3	Oilers (W)	88	41	-6	0
'05-'06	Lightning (E)	92	43	1	Canucks (W)	92	42	0	-1

^aSeason shortened to 48 games by a lockout.

each conference, which can be considered wild-card teams, are the two remaining teams with the most points. Thus, it is possible to have five teams qualify for the playoffs from one Eastern (or Western) division, and only three qualify from the other Eastern (Western) division.

3 Applying the new rules to recent seasons

If half of the Eastern Conference teams (8 out of 16) make the playoffs in any particular season, while more than half of the Western Conference teams (8 out of 14) qualify, it stands to reason that the 8th seed in the East will be a better quality team than the 8th seed in the West. As a first test of this reasoning, we apply the rule changes to the seven NHL seasons that have occurred since the 2004–2005 lockout.⁴ To do this, we imagine that Winnipeg, Detroit, and Columbus have been moved to their new conferences, and the league has been reorganized into the new four-division structure. We then apply the new playoff qualification rule to the final standings.⁵ The results of this thought experiment can be found in Table 2. The *Removed from Playoffs* block of the table provides

details about the teams that made the playoffs in real life, but would not have qualified under the new rules. The *Added to Playoffs* block provides details about the teams that did not qualify, but would have made the playoffs if the new rules had been in place.

Several things stand out from this table. Most importantly, in every season except 2007–2008 the new alignment would have resulted in a different set of 16 teams qualifying for the playoffs than occurred in reality. The playoff wins column shows that most of the “removed” teams did not make it out of the first round of the playoffs. The major exception is the 2009–2010 Montreal Canadiens, who won nine games and advanced to the Eastern Conference Finals.

The other important takeaway from the table is that every team removed from the playoff picture comes from the new Eastern Conference, and every team that added to the playoff picture comes from the new Western Conference. This provides some evidence that it will be easier to qualify for the playoffs in the West than in the East under the new rules. Additionally, the *Diff. In* block shows that in five of eight instances, the team added to the playoffs had fewer wins than the team they would have replaced, and in five instances they had fewer points.

Table 2 is not conclusive evidence that the new realignment rules will unfairly favor Western teams' playoff chances over Eastern teams, but there are several take-away points that should be emphasized. The empirical evidence indicates that in most seasons application of the new playoff qualification rules would have benefited a team in the Western Conference and hurt a team in the Eastern Conference. Additionally, in most cases, the new rules resulted in giving a playoff spot to a team with a worse regular season record than the team they were replacing. Certainly, a few years of data is not enough to

⁴ We restrict the analysis to these seasons because significant rule changes were made following the 2004–2005 lockout. Chief among these changes was the introduction of the shootout, which has inflated win and point totals.

⁵ This approach does not take into account the new scheduling matrix, nor the almost inevitable changes to total points earned by each team. This section is merely a thought experiment which sets up the remainder of the paper, which account for changes to the scheduling matrix.

make any broad conclusions about the fairness or unfairness of the new system, so in the remainder of the paper we use Monte Carlo simulations to make statistical inferences about whether the conference gap exists, and how big it is.

4 Simulation of full NHL seasons

In order to take into account all the aspects of the new NHL realignment and rule changes, we use a Monte Carlo simulation strategy.⁶ The basic intuition behind the model is that each of the 30 teams has an underlying level of talent or skill, and in each game the teams perform at a level near, but not necessarily exactly equal to, their talent level. The outcome of each game is a function of the two teams' performance levels, where the team with the better performance wins the game.

4.1 The model

The simulation model has three key parameters: the probability that the worst team in the league beats the best team in a random game (p), the standard deviation in the game-to-game performance levels for each team (τ), and the standard deviation in the team talent levels (σ).⁷ It's important to note that the researcher must choose, a priori, a value for either p or a value for the ratio of τ to σ ,⁸ which are related as follows:⁹

$$\frac{2.8854}{\Phi^{-1}(p)} = \frac{\tau}{\sigma}.$$

We begin the simulation by drawing a “team quality” value for all thirty teams, μ_i , from a normal distribution with mean zero and variance σ^2 :

$$\mu_i \sim N(0, \sigma^2).$$

⁶ Buttrey, Washburn and Price (2011) simulate outcomes of NHL games as a function of independent Poisson processes of goal scoring because they are interested in studying the arrival rate of goals in hockey. Since we are only concerned with winner and loser of each game, we opt for a simpler algorithm which requires fewer modeling assumptions.

⁷ σ parameter can be thought of as a measure of how much parity there is in the league. Values of σ close to zero indicate that there is high parity and all teams have the same level of talent.

⁸ It is only the ratio of these two values which matters. Setting $\tau = \sigma = 1$ yields the same results as $\tau = \sigma = 100$.

⁹ See the Appendix for a full proof of this relationship.

You can interpret team i 's talent as a unitless, mean-zero measure of the underlying talent of team i . The μ_i 's are held constant through the entire season.

After setting the talent levels for all 30 teams, we simulate a full season of 1230 games. Matchups for these games are set by the scheduling formula the NHL uses, which is described in Table 1.¹⁰ In each game a team might play slightly better or worse than their talent would suggest. To account for this, a “game performance” value is drawn for the home and away teams. These values, γ_i and γ_j , are independently simulated from normal distributions with means equal to μ_i and μ_j and variances equal to τ^2 :¹¹

$$\begin{aligned}\gamma_i &\sim N(\mu_i, \tau^2) \\ \gamma_j &\sim N(\mu_j, \tau^2).\end{aligned}$$

The winner of each game is then determined by comparing the values of γ_i and γ_j . In most games, the team that has a higher γ (i.e., the team that plays better) wins the game and receives two points in the standings. Over the past 15 seasons, about 22.4% of NHL games went into overtime. Because teams that lose in overtime receive one point in the standings, we must account for this in the simulation. To determine which simulated games go to overtime, we calculate a tie threshold, α , such that:¹²

$$Pr(|\gamma_i - \gamma_j| < \alpha) = 22.4\%.$$

For games in which $|\gamma_i - \gamma_j| \geq \alpha$, two points are awarded to the team with a higher value of γ and zero points are awarded to the other team. Games in which $|\gamma_i - \gamma_j| < \alpha$ are deemed to have gone to overtime, and each team receives one point in the standings. A second point is awarded to the team that wins the game in overtime. To simulate the outcome of overtime, first we linearly rescale the game performance values so that they range from zero to one:

$$\zeta_{ij} = \frac{(\gamma_i - \gamma_j) - a}{b}$$

where a and b are the mean and maximum of all the $\gamma_i - \gamma_j$'s, respectively. The overtime winner, and recipient

¹⁰ Each season is simulated twice, once using the new scheduling formula and playoff qualification rules, and once using the old formula and rules.

¹¹ Although we could have team-specific values of τ^2 , we make the simplifying assumption that $\tau_i^2 = \tau^2 \forall i$. In other words all teams have the same variability in game performance, regardless of their underlying quality.

¹² Calculating α this way guarantees that the total number of points earned by all teams in each simulated season is approximately equal to the total points earned in real NHL seasons.

of the second point in the standings, is determined by a weighted coin-flip, where the probability of the home team winning, ζ_{ij} , is proportional to the difference in game performances:

$$OTwinner_{ij} \sim \text{Bern}(\zeta_{ij}).$$

After all game outcomes have been simulated and points awarded, the final standings, playoff teams, and conference gap is determined based on the new or old division structure and playoff qualification rules. We then draw new values of μ_i 's and repeat until we have simulated 10,000 seasons under both the new and old rule regimes.

4.2 Properties of the model

There are a few aspects of the simulation strategy which merit further elaboration. First, we will show in the robustness section that the ratio of τ to σ is theoretically unbounded, but small values of the ratio imply unrealistic values of p . In the main results of the paper, we chose to set $p=0.25$ because in each season since the NHL expanded to 30 teams in 1999–2000, the team with the worst record in the league beat the team with the best record 25% of the time.¹³ This implies that $\tau/\sigma=4.28$, or that there is about four times the variability in the game performances than in the team talent levels.¹⁴

Also when we determine the outcome of overtime periods, we assume that a team that is playing better hockey (i.e., the one with the higher “game performance” value for that game) is more likely to win the game. Alternatively, we could have assumed that because of the sudden-death nature of overtime, each team enters the period with a 50–50 chance of winning. Rosenfeld et al. (2010) show that in the NBA overtimes tend to be won by the team performing better overall, while NFL overtimes tend to be more random. Unfortunately their research does not comment on the tendency in the NHL. Later in the paper, we show that this modeling assumption does not affect the substantive takeaways from this simulations.

It is also crucial to emphasize that none of the simulations take into account which teams are good or bad in real life. The team numbers are arbitrary placeholders. This approach sets aside any subjective concerns about

one conference having more historically successful teams than the other conference. The question we are trying to answer is whether the new NHL alignment would be a fair configuration if all 30 teams had an equal chance of being very good or very bad. Put another way, if the NHL ignored geography and randomly placed teams into four divisions, would the teams in the 7-team divisions have an advantage over the teams in the 8-team divisions?

5 The conference gap in 10,000 simulated seasons

The results of the Monte Carlo simulations of 10,000 full NHL seasons are presented in Figure 1. The red lines and text correspond with the old alignment and rules; the blue lines and text correspond with the new alignment and rules. As the graph shows, the mean conference gap under the old alignment is not statistically significantly different from zero (-0.04 ; SE: 0.05), while the mean conference gap under the new alignment is 2.74 points (SE: 0.06), which is statistically significantly different from zero ($p<0.001$). Also, the old alignment does not seem to favor one conference over the other in terms of the percentage of time that one conference's eighth seed has more points than the other conference's eighth seed. This is not the case, however, with the new alignment in which the East's 8th seed has more points than the West's 8th seed 62.5% of the time.

The simulation strategy allows for the calculation of other interesting quantities. Table 3 presents the proportion of the simulations in which certain non-playoff teams would have made the playoffs had they been in the other conference. In other words, in what proportion of simulated seasons did the West's 9th place team have enough points to make the playoffs had they been in the East? This is perhaps the most relevant question of all, since the conference gap between 8th seeds is irrelevant if there is not a strong 9th seed to replace a weaker 8th. Under equitable playoff rules, the 9th best team in one conference will have a better record than the 8th best in the other conference some of the time. If the league is structured fairly, this unlucky 9th place team should be equally likely to come from the Eastern or Western Conference.

The “Old Alignment” column of Table 3 indicates that with the old rules lower seeds in both the East or West would have benefited from being in the other conference at similar rates. About 29% of the time, the best non-playoff team would have made the playoffs had they

¹³ The best team beat the worst 30 times, lost in regulation 8 times, lost in overtime once, lost in a shootout once, and tied once.

¹⁴ Because only the ratio matters, we anchor τ at 1 and use $\sigma=0.23$ in the main simulations.

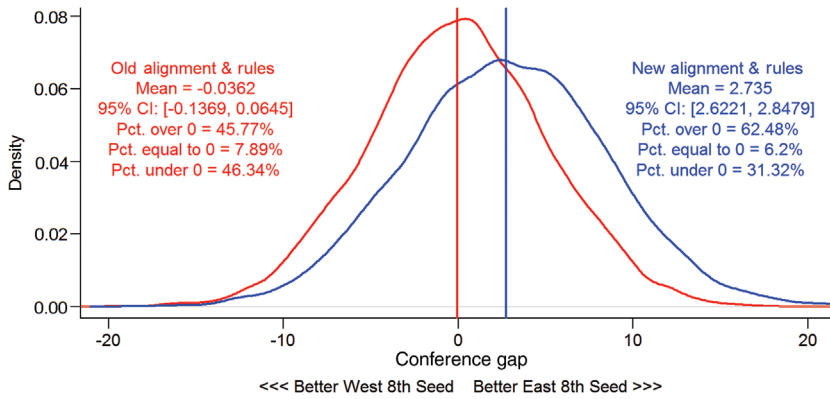


Figure 1 Simulation of 10,000 full NHL seasons.

Table 3 Percentage of time non-playoff teams would have made playoffs in other conference.

	New alignment	Old alignment
9th Seed		
East	38.6%	28.6%
West	20.8%	29.0%
10th Seed		
East	21.1%	15.4%
West	8.6%	15.4%
11th Seed		
East	9.7%	7.0%
West	2.7%	7.0%

been in the other conference.¹⁵ This number is about 15% for the second best non-playoff teams. The “New Alignment” column suggests that such balance will not occur with the new rules. Eastern Conference 9th seeds would have made the playoffs 39% of the time if they were in the West. That is almost twice as often that of Western Conference 9th seeds, who would make the playoffs only 20.8% of the time if they were in the East. For 10th seeds, this percentage is more than double in the East as in the West. For 11th seeds, it is almost four times as much.

Another way to conceptualize this result is by imagining that we choose five random seasons played under the new alignment. The Monte Carlo results presented here suggest that in two of those seasons the 16 best teams will make the playoffs. In another two of those seasons, a team in the West will make the playoffs with a worse record

than a non-playoff team in the East. The reverse will be true in just one of the five seasons.

6 Validity and robustness of simulation results

In this section we explore how closely the simulated seasons match real-world data and demonstrate the robustness of the results to modeling assumptions. Figure 2 compares simulation distributions of points to data from real NHL seasons. The figure sorts the 30 point totals from smallest to largest within each season and plots the density of the number of points earned by the best team each year under the new rules.¹⁶ For example, the top boxplot represents the number of points earned by the 30th best (i.e., the worst) team in each of the 10,000 simulations. The endpoints of the whiskers are the minimum and maximum points earned by the worst team. The sides of the box represent the 2.5% and 97.5% quantiles. The red dots indicate the number of points earned by the 30th best team in each full NHL season since the 2004/2005 lockout. If our simulations are doing a good job of generating realistic looking standings, these red dots should lie within the boxes. This is the case for almost all the plots in the figure, particularly for teams in the middle of the standings. These are the teams for whom it is most important to simulate realistic point totals, since they are the ones right at the margin of making or not making the playoffs and are used for calculating the conference gap.

¹⁵ The differences in this column are due to random variation and do not provide evidence for any underlying trends under the old alignment.

¹⁶ The figure looks almost identical when using the results from the old rules.

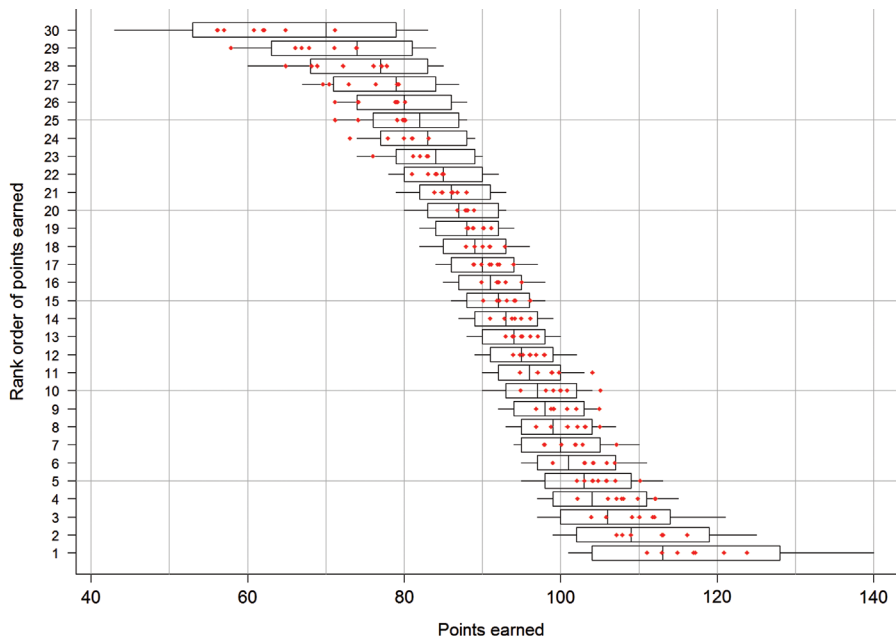


Figure 2 These Box-and-Whisker plots show the total number of points earned by the i^{th} best team in each of the 10,000 simulations. The boxes represent the 2.5%, 50%, and 97.5% quantiles. The endpoints of the Whiskers show the minimum and maximum points earned by the i^{th} best team. The dots signify the number of points earned by the i^{th} best team in each full NHL season since the 2004/2005 lockout.

The results from the simulations are also robust to the assumptions made about p , τ , and σ . Figure 3 shows estimated conference gaps based on 1000 simulations using different values of the τ/σ ratio. The curved line shows the value of p that corresponds with each value of the σ/τ ratio. Recall that p was set to 0.25 because this has been

the empirical probability in recent years and the ratio was calculated to be 4.28. It is clear from the figure that the estimated conference gap does not deviate from 2.74 to a statistically significant extent as the ratio σ/τ increases. In other words, when there is more between-game variation in each team’s game performances (τ) relative to

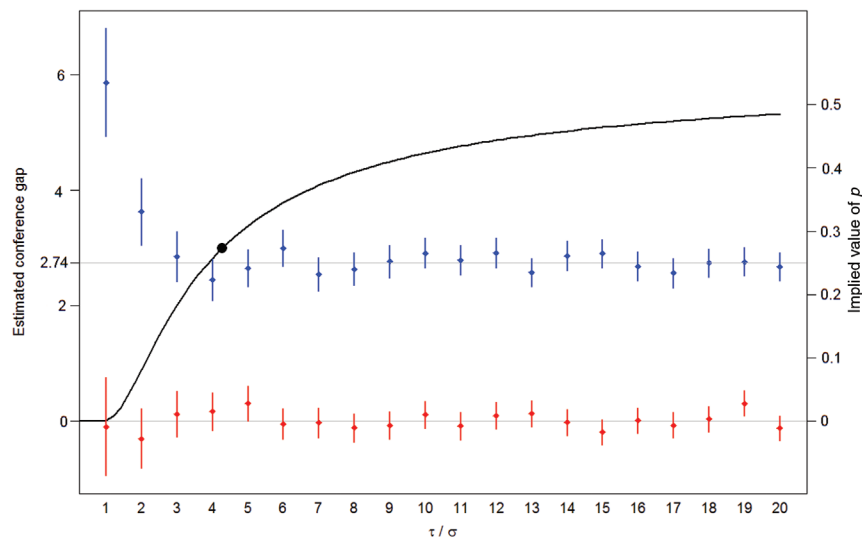


Figure 3 The bars represent the conference gap given different values of σ/τ . The lower set of bars are from simulations under the old alignment; the upper set are from simulations under the new alignment. The curved line represents the relationship between p and σ/τ that is implicit in the simulation. The large dot indicates the empirical estimate of p (0.25) and the corresponding ratio σ/τ (4.27) used in the simulation. Recall that this combination of parameters yielded an estimated conference gap of 2.74.

the variation in the talent of the league (σ), the size of the conference gap stays between 2 and 3. At the limit where all teams have exactly the same underlying talent (i.e., as $\sigma \rightarrow 0$) the new realignment structure will continue to be biased against Eastern Conference teams.

The other trend to notice is what happens as the ratio approaches zero. In this scenario teams play consistently from game-to-game, but there is a very high variance in team quality. In such a world upsets would be rare, the best team would win nearly all their games, and the worst team would lose nearly all their games. In this scenario, the conference gap increases dramatically. When there is 1000 times more variance in the team qualities, σ , than team performances, τ (not presented here), the size of the conference gap is about 10 points.

While the results on the left of Figure 3 imply a larger conference gap, they should not cast doubt on the validity of the estimate of the gap at 2.74. This is because low values of the ratio are not realistic. When τ is <2.25 times larger than σ the probability of the worst team beating the best is <10%. When there is equal variation in team quality and between-game performances ($\tau = \sigma$), there is just a 0.2% chance of such an upset. Given that over the previous decade and half the worst team has prevailed in 25% of these matchups, it is reasonable to dismiss the left-most conference gap estimates as being generated from an NHL with a competitiveness structure that is very dissimilar from reality. At worst, we can treat 2.74 as robust estimate of the lower bound of the conference gap.

The another key takeaway is perhaps more concerning. If we treat p as a measure of the degree of parity in the NHL, then Figure 3 suggests that increasing parity in the league will not eliminate the conference gap bias in favor of Western Conference teams. For numerous reasons, the NHL should value parity among its teams. Yet if all thirty teams had virtually identical levels of skill and talent, the teams in the West would have a much higher probability of making the playoffs. In fact, when you simulate the model, giving all 30 teams the exact same level of average team quality, the 8th seeded Eastern Conference team has roughly 2.9 more points than the 8th seeded Western Conference team, on average.

Another important assumption embedded in the model is how we treat overtime outcomes. The results presented so far assume that teams' performances in overtime will be similar to their performances in regulation. In other words, two teams with game performance values, γ_j 's, that are nearly identical to each other will each have a 50% chance of winning in overtime. Alternatively, if two teams have γ_j 's that put them just inside the tie threshold, α , the team with the higher γ_j is very likely to win in overtime.

6.1 Alternative models

It is possible that once a game goes to overtime each team's winning probability is about 50%. This belief can be incorporated into the Monte Carlo model by drawing overtime results from a Bern(0.5) distribution instead of a Bernoulli(ζ_{ij}) distribution.¹⁷ Changing this assumption has no affect on the substantive takeaways from the simulation model. Under the Bern(ζ_{ij}) assumption, the average conference gap in the new alignment is 2.74; with an assumption of Bern(0.5) the conference gap is estimated to be 2.75 (SE: 0.07). It is not surprising that this assumption does not affect the conference gap, since affecting it would require making an assumption that overtimes occur differently in the Western Conference than in the Eastern.

As a separate check to the validity of our results, we estimated the conference gap based on games simulated using Bill James's log5 method (sometimes called the Bradley Terry model) (Albert and Bennett 2001). We know from empirical data since the 2004/2005 NHL lockout that team winning percentages have a mean of 0.5 and a variance of 0.00825. To simulate a season under this alternative model, we drew 30 expected winning percentage values from a beta distribution with that mean and variance.¹⁸ Then for each of the 1230 games we calculate the probability that team A wins based on the formula

$$p_{A,B} = \frac{p_A(1-p_B)}{p_A(1-p_B) + p_B(1-p_A)}$$

where p_A and p_B are the expected winning percentages of the two teams in that game. We then take a single draw from a $Bern(p_{A,B})$ distribution to determine the winner. We then calculate the standings and conference gap for the season.¹⁹

Simulating 10,000 seasons using this technique yields results which are virtually identical to those from the earlier model. Under the new alignment the conference gap is estimated to be 2.80 points (SE: 0.07), which is statistically indistinguishable from the original result of 2.74 points (SE: 0.06). As before, with the old alignment, the estimated conference gap is not statistically different from zero (mean: -0.09; SE: 0.07). Also before, the proportion of seasons in the conference gap benefits the West is

¹⁷ Where ζ_{ij} is the rescaled value of $\gamma_i - \gamma_j$.

¹⁸ We used the beta distribution because it guarantees that the expected winning percentages are bounded by zero and one.

¹⁹ Overtimes were treated in a similar way as the original model. A tie threshold (on the scale of $p_{A,B}$) was calculated, and games within that threshold were deemed as having gone to overtime.

62.9%, whereas the proportion in which the East benefits is just 31.7%. These results provide even more evidence that the conclusions from the original simulation model are valid and that we should expect the average conference gap to be between 2.5 and 3 points.

7 Discussion

7.1 Possible critiques

Two criticisms to our findings might be raised, although neither lessens the implications of the results. First, one may point out that Western Conference teams have, in recent years, been of higher quality than Eastern Conference teams. While this may be true today, there is no reason to think that this will still be the case a few years from now. The Monte Carlo results show that if the conferences had equally talented teams, teams in the West would be better off in terms of reaching the playoffs. Also, the far right of Figure 3 suggests that if all thirty teams had exactly the same amount of underlying talent, the West would still have a realignment advantage.

A second criticism is that teams in the West are already at a disadvantage because of more grueling travel schedules and providing them with an advantage via the realignment is only fair (Smith 2013). If there is indeed a travel effect, Western teams would tend to have worse road records and better home records, since their home opponents would be suffering from travel fatigue themselves. Because every team has an equal number of home and away games, a travel fatigue effect could not have as large a systematic effect as the conference gap.

In this paper, we have brought to light potential unintended consequences of NHL realignment and changes to the rules for playoff qualification. Using tens of thousands simulated seasons, we have found robust evidence that the NHL's new structure will unfairly benefit teams in the Western Conference over teams in the Eastern Conference. We should expect that on average the threshold for reaching the playoffs in the East will be 2.74 points (SE: 0.06) higher than the threshold for reaching the playoffs in the West. This difference is of critical importance. In the lockout-shortened 2013 season, only two points separated the East's 5th seed from the 11th seed. In the 2010–2011 season, three extra points would have put Calgary in the playoffs. Instead they finished in 10th place. In 2008–2009, the 9th seeds in the East and West would have moved up to 7th and 6th place (respectively) with three extra points. Indeed in the single season played under the

new alignment, the size of the conference gap was two points. To be sure, one season cannot prove or disprove the hypothesis, but the fact that this empirical evidence is in line with the findings here is suggestive.

The implications of these results are far reaching. For owners and team executives, it means imbalances in the revenue earned from home playoff games. Western Conference teams will make the playoffs at higher rates than Eastern Conference teams, meaning at least two extra games' worth of ticket and concession sales. For players, it means that playing for a Western Conference team gives them a better chance of winning the Stanley Cup in any given year, since just making it to the playoffs gives them a chance to win it all.²⁰ For fans of Eastern Conference teams, it means a higher probability that their season will end too soon and less of a chance that in any given year his or her team will win the Stanley Cup.

Appendix

Recall that p is the probability that the worst team beats the best team in a random game. τ^2 is the variance in the game performances for each team. σ^2 is the variance of underlying team talent. We can rewrite the value of p as:

$$\begin{aligned} P(\gamma_{\text{worst}} > \gamma_{\text{best}}) &= p \\ P(\gamma_w - \gamma_b > 0) &= p. \end{aligned}$$

Now for clarity of notation, define $d_{w,b}$:

$$\gamma_w - \gamma_b \equiv d_{w,b}.$$

Because both γ 's have a normal distribution with the same variance:

$$d_{w,b} \sim N(\mu_w - \mu_b, 2\tau^2).$$

Writing p in terms of $d_{w,b}$:

$$\begin{aligned} P(d_{w,b} > 0) &= p \\ P(d_{w,b} - (\mu_w - \mu_b) > -(\mu_w - \mu_b)) &= p \\ P\left(\frac{d_{w,b} - (\mu_w - \mu_b)}{\sqrt{2\tau^2}} > -\frac{\mu_w - \mu_b}{\sqrt{2\tau^2}}\right) &= p. \end{aligned}$$

²⁰ As the 8th seeded Los Angeles Kings demonstrated in the 2012 playoffs.

And

$$\begin{aligned} \frac{d_{w,b} - (\mu_w - \mu_b)}{\sqrt{2\tau^2}} &\sim N(0, 1) \\ P\left(Z > \frac{\mu_b - \mu_w}{\sqrt{2\tau^2}}\right) &= p \\ 1 - P\left(Z \leq \frac{\mu_b - \mu_w}{\sqrt{2\tau^2}}\right) &= p \\ 1 - \Phi\left(\frac{\mu_b - \mu_w}{\sqrt{2\tau^2}}\right) &= p \\ \Phi\left(\frac{\mu_b - \mu_w}{\sqrt{2\tau^2}}\right) &= (1 - p) \\ \frac{\mu_b - \mu_w}{\sqrt{2\tau^2}} &= \Phi^{-1}(1 - p). \end{aligned}$$

Because of the symmetry of the normal distribution:

$$\begin{aligned} \frac{\mu_b - \mu_w}{\sqrt{2\tau^2}} &= -\Phi^{-1}(p) \\ \mu_b - \mu_w &= -\sqrt{2\tau^2} \Phi^{-1}(p) \\ E(\mu_b | \sigma) - E(\mu_w | \sigma) &= -E(\sqrt{2\tau^2} \Phi^{-1}(p) | \sigma) \\ E(\mu_b | \sigma) - E(\mu_w | \sigma) &= -\sqrt{2\tau^2} \Phi^{-1}(p). \end{aligned}$$

Because fix the mean of the team quality distribution to be zero, then by the symmetry of the normal distribution:

$$-E(\mu_w | \sigma) = E(\mu_b | \sigma).$$

Therefore:

$$-2E(\mu_w | \sigma) = -\sqrt{2\tau^2} \cdot \Phi^{-1}(p).$$

And by definition, $E(\mu_w | \sigma)$, is the thirtieth order statistic in thirty draws from the team quality distribution, $N(0, \sigma)$. The expected value of order statistics from a normal distribution (Royston 1982) is:²¹

$$E(r, n) = \mu - \Phi^{-1}\left(\frac{r - \alpha}{n - 2\alpha + 1}\right) \cdot \sigma$$

where n is the number of samples drawn, r is the order statistic of interest, μ is the mean of the distribution, σ is the standard deviation of the distribution, and α is an ancillary parameter which Royston recommends be set to 0.375.²² Therefore:

²¹ This is the formula for the approximation of the expected value. It is accurate to within 0.001.

²² Simulating this expected value confirms the result here.

$$\begin{aligned} E(\mu_w) &= 0 - \Phi^{-1}\left(\frac{30 - 0.375}{30 - 2 \times 0.375 + 1}\right) \cdot \sigma \\ E(\mu_w) &= -2.0403\sigma. \end{aligned}$$

Now substituting back into the original equation:

$$\begin{aligned} -2 \times -2.0403\sigma &= -\sqrt{2\tau^2} \cdot \Phi^{-1}(p) \\ \frac{2.8854}{\Phi^{-1}(p)} &= \frac{\tau}{\sigma}. \end{aligned}$$

In the main results of the paper we assume that $p = 0.25$, so

$$\frac{\tau}{\sigma} = 4.2779.$$

Because the results of the Monte Carlo model are only affected by the relative size of τ and σ and not their absolute magnitudes, we can anchor τ at 1. Therefore

$$\sigma = 0.2338.$$

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